Brightness matching by peripheral vision of motion

Gerald Klaus Kranewitter

Mathematisches Institut der Universität Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria

Roman Rudolf Liedl

Mathematisches Institut der Universität Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria

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In this paper we introduce a method (called teeming-photometry), which can be used better than the flicker-photometry to specify the brightness of color stimuli's that are presented periodically in time like on TV and computer monitor. Hereby the subjective judgment of brightness flickering occurring in flicker photometry is replaced by an observed decision of motion. Instead of foveal observing flickering stimuli's our method uses parafoveal and peripheral observation of apparent and randomised motion. There is also the possibility to match the average brightness of spatial inhomogeneous stimuli, which show a spatial double periodical pattern. Such in several respects teeming photometry expands the application range of flicker photometry.

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Introduction

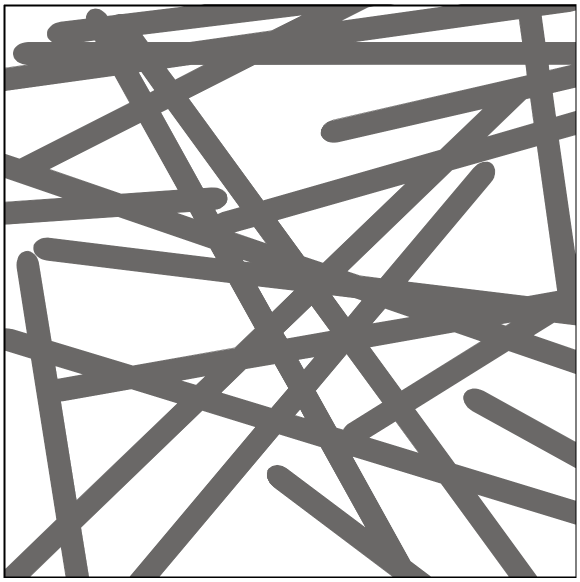
There are numerous older[1] and newer[2,3,4,55], statically and dynamical approaches for experimental brightness assignation to light emitting areas.
This methods are adapted to various purposes. For brightness matching the most important method is the classical flicker photometry. It was invented by Odgen Nicholas Rood in 1893 and it is the experimental groundwork for the definition of the luminous efficiency functions[56,1].
Each method is based on a confrontation of a reference component and a component in question that should be rated.
Some methods are based on a direct statically comparison of two color stimuli. Other methods use visual effects, which occur in more complex situations, where for instance stimuli in spatial or temporary inhomogeneous compounds are presented to the observer. Basically each method is creating a specific brightness definition. But comparing the results within the reference stimuli of the miscellaneous methods there appear only rather slight differences and all methods are well compatible with the methode of direct heterochromatic brightness matching[5].
Therefore we expect results in this dimension also for a methode which assignes a brightness to stimuli which are periodical in time but are spatial homogenously. Obviously this is for frequencies that are higher than the flicker fusion frequency (20-60Hz). Only for lower frequencies (not part of our paper) the Broca-Sulzer-effect[6] is prominent and this effect leads to a considerable increasing of the perceived brightness.

The methode introduced in this paper - we call it teeming photometry (there is an affinity to the visual impression of „teeming with rain“) - is dedicated to the brightness measurement of temporal periodical stimuli. In common statical methodes for brightness measuring require a matching which is diffucult for the observer and therefore the results are rather less reproducible and less faithful. On flicker-photometry – a dynamical methode – matching is easier and more reproducible. But disturbing stroboscopic side effects[7,8] perform if the stimulus is periodical in time (on TV and computer screens).
Therefore we want to present a dynamical methode which allowes a fashionable and proper brightness matching of stimuli which are periodical in time.

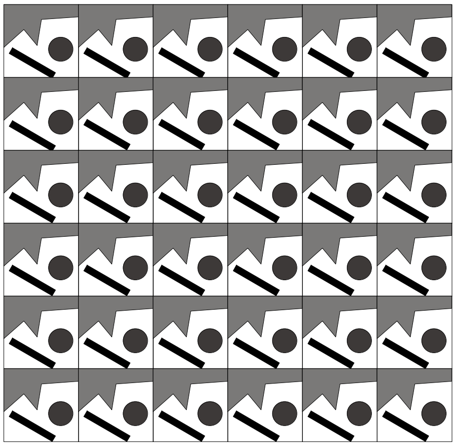
Methods

Our experiments have been realised with 40 observers (male and female, 21 till 74 yearws old, testes on color efficiency by a Nagel-anomaloscope). One observer was deuteranope, and an other protanormal. With regard to the ethnic provenience we had 2 Nepaleses, 1 Chinese, 1 black african from Ruanda, 1 Japanese, 1 Philippine and a rest of Caucasian. Each observer had at least college edjucation but only the two autors of this paper have been familiar with the intension of the experiments. Wearers of spectacles could use them. The non caucasians observers were found by a universitary procuration. The other observers have been acquaintances.
The rectangular stimulus (20 cm in length and 10 cm in hight) is produced in the center of a 19-inch CRT-monitor with a refresh rate of 100 Hz. On the screen there is still a small control field and a black rest area. The complete stimulus consists of a background and a foreground. In the first experiment the background is uniformely colored by a unique RGB-color. The differences of the color on the background coming from the inhomogenity of the monitor have been negligible. The foreground is build from the reference stimulus. This is a pattern of 40 straight lines with width 4 mm and endpoints inside of the background rectangle chosen by a random generator. In one seconde there are given 18 such different line patterns and therefore one pattern

is presented for about 50 ms. The color of the reference stimulus can be chosen in a scale of 255 grey colors between black and white. A typical pattern of the stimulus looks like shown in the following picture.



The distance between observer and monitor face is between 50 and 70 cm. Such the width of the grey lines is about 0.3 and 0.45 degree viewing angle. But the view of the observer is fixed on a point outside of the monitor for achieving parafoveal or peripheral observation of the stimulus.
The observer was instructed to start with peripheral viewing of ca 30^o to match the intensity of the reference grey into the perception of a minimal apparent motion of the lines. Now the question is to increase the angle for peripheral observation until all intensities of the lines grey lead to a complete standstill of the apparent motion (this will occur especial for red and for blue backgrounds). On the other hand, if there is a quite interval of the lines grey intensity, for which there is no apparent motion to see, then the angle of peripheral observation has to be decreased (primary for green backgrounds). So in any case of background the observer has to find a point outside of the monitor, such that gazing this point the minimal apparent motion is occurring only with one intensity of the grey lines. Summarising for saturated red or blue backgrounds there has be used an angle of 50-70 degrees and for green, yellow, magenta and cyan background colors there may suffice an angle of 20-30 degrees peripheral vision to inhibit the apparent motion of the grey lines for a unique intensity of the grey color.
The experiments can be implemented in normal offices at different daytimes with natural or artificial illumination and also in a shaded laboratory.
In our experiment the observers had to match 24 relatively saturated monitor colors, that are colors with a RGB characterzation (R, G, B), where Min{R,G,B}=0 and Max {R,G,B}=255.
A smaller sample of only 5 observers had to match the 343 monitor colors with RGB characterzation (R, G, B), where $R, G, B \in \{0, 1*42, 2*42, \dots , 5*42, 255\}$.
For comparing the results we used a computer simulated flicker photometer.
In a further experiments the grey line pattern was replaced by uniformly colored line patterns. And finally the uniformly colored backgrounds where replaced by spatial double periodic patterns which had contrasts in luminance, hue and saturation.



The results have been saved automatically and where discussed with interested observers at most after finishing their test series.

Now (speaking in terms of the RGB-characterization) the teeming experiment yields by matching for each monitor color F=(R, G, B) (used as background color) with $0 \le R, G, B \le 255$ to a grey color (H_F, H_F, H_F) (used as color of the grey lines) with a personally depending value of H_F such that $0 \le H_F \le 255$. Then the "teeming brightness" B(F) of the color F is defined to be the luminance Y_F of the grey color H_F. For instance if the Gamma of the monitor is chosen with 2,38 and if the luminance of the white monitor color (255, 255, 255) is 87,6 cd/cm^2, so we can calculate the luminance

$$B(F)=87{,}6*(H_F/255)^{2{,}38} \text{ cd/m}^2 \qquad (1)$$

or this luminance can be measured directly by a photometer.

Accuracy of measurement.

By the computer program the observer was ordered to find a natural number $k \in \{0, 1, ..., 51\}$ such that the apparent motion of the grey line pattern characterized with (R, G, B) = (5k, 5k, 5k) on the monitor with Gamma=2,38 had a smaller amount than the grey line patterns characterized by (5k-5, 5k-5, 5k-5) and by (5k+5, 5k+5, 5k+5). This task was reproducibly solvable for almost every matching and therefore the observer always found a unique result. Supposing that there exists a minimum of apparent motion for a unique grey (ρ, ρ, ρ) we have $5k-5 < \rho < 5k+5$ but experiments have shown, that $5k-3 < \rho < 5k+3$ holds anyway.

Therefore $|5k-\rho| < 3$. Such the error for our method of matching is smaller than 3.

Using formula (1) we get for $252 < \rho < 255$ an error smaller than

$$87{,}6*(255/255)^{2{,}38} \text{ cd/m}^2 - 87{,}6*(252/255)^{2{,}38} \text{ cd/m}^2 = 2{,}43 \text{ cd/m}^2. \qquad (2)$$

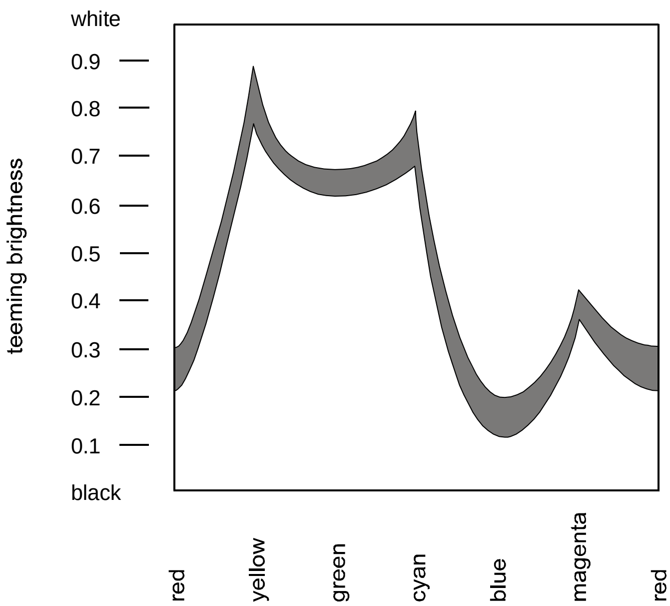
This is about 2,7% of 87,6 cd/m^2. For smaller amounts of ρ we get obviously a smaller absolute error and a bigger relative error.

Results

The results of our experiments manifest no dependence from the age, the sex, the ethnic origin and the color efficiency of the observers. We could not find any dependence from the ambience and the situation of the illumination of the room. Furthermore the results for a big part of the observers where close together. The repetition of the experiments for some observers after the time of 5-10 months shows a surprising constancy of the results.

Using uniformly colored backgrounds of the stimulus the comparison with the computer simulated flicker-photometry could show no differences to the teeming-photometry because of the unstableness of the flicker-photometry.

In the following diagram we can see the teeming-brightness of the 24 relative saturated monitor colors detected by the 40 observers.



Considering only one observer P and describing any monitor color F by the CIE-characterzation X_F, Y_F, Z_F it is possible to find real coefficients ξ_P, ψ_P, ζ_P such that the teeming brightness B(F) can be expressed (within the accuracy of measurement) as a linear function

$$B(F) = \xi_{P*}X_F + \psi_{P*}Y_F + \zeta_{P*}Z_F \quad (3)$$

where the coefficients ξ_P, ψ_P, ζ_P are independent from the color F and attributed only to the observer P.

The primaries $F_R = (255, 0, 0)$, $F_G = (0, 255, 0)$, $F_B = (0, 0, 255)$ (in R,G,B characterzation) of the monitor form a basis of the CIE-linear color space und therefore it suffices to determine their teeming brightness's $B(F_R)$, $B(F_G)$, $B(F_B)$ in order to calculate the coefficients ξ_P, ψ_P, ζ_P.

If we assume, that the CIE-characterzations of F_R, F_G, F_B are (X_R, Y_R, Z_R), (X_G, Y_G, Z_G), (X_B, Y_B, Z_B) and the associated teeming brightnesses are $B(F_R)$, $B(F_G)$, $B(F_B)$

then we have

$$B(F_R) = \xi_{P*}X_R + \psi_{P*}Y_R + \zeta_{P*}Z_R, \quad (4a)$$

$$B(F_G) = \xi_{P*}X_G + \psi_{P*}Y_G + \zeta_{P*}Z_G, \quad (4b)$$

$$B(F_B) = \xi_{P*}X_B + \psi_{P*}Y_B + \zeta_{P*}Z_B \quad (4c)$$

and from this system of three linear equations we get uniquely the coefficients ξ_P, ψ_P, ζ_P.

As an example we have measured for our CRT-Monitor $(X_R, Y_R, Z_R) = (49.49, 26.96, 3.36)$, $(X_G, Y_G, Z_G) = (29.94, 63.06, 11.42)$, $(X_B, Y_B, Z_B) = (13.58, 6.79, 71.30)$. For the first named author

K we got by teeming photometry ($B(F_R)$, $B(F_G)$, $B(F_B)$) = (21.40, 61.25, 13.00). The solution of the linear system yields to (ξ_K, ψ_K, ζ_K) = (-0.1258, 1.011, 0.11). But for the second named author L we had ($B(F_R)$, $B(F_G)$, $B(F_B)$) = (26.10, 63.20, 9.60) and therefore the coefficients are (ξ_L, ψ_L, ζ_L) = (-0.0233, 1.005, 0.0436). These second result was exceptionally close to (0,1,0) and therefore for each monitor color F with the CIE-characterzation F=(X, Y, Z) there holds $B(F) \approx Y$ for the second named author L. The results for the first named author K are rather typical for the bigger part of our observers.

Now we have an occurrence of overdetermination.

For the white monitor color W = (255, 255, 255) with CIE-characterzation (X_W, Y_W, Z_W) at the one hand the calculation with the linear formula leads to $B(W) = \xi_P * X_W + \psi_P * Y_W + \zeta_P * Z_W$ but at the other hand by teeming photometry obviously we get $B(W) = Y_W$. For eliminating this contradiction we calculate for each observer P a factor κ_P by

$$\kappa_P = Y_W / (\xi_P * X_W + \psi_P * Y_W + \zeta_P * Z_W). \qquad (5)$$

Then we replace the coefficients ξ_P, ψ_P, ζ_P by $\kappa_P * \xi_P$, $\kappa_P * \psi_P$, $\kappa_P * \zeta_P$. We found that this manipulation is possible within the accuracy of measurement.

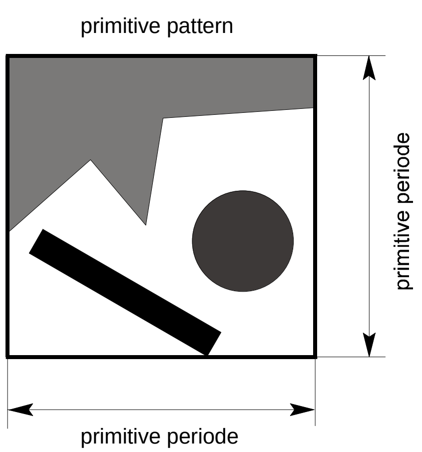
Using this mended coefficients for association a black and white picture to a color picture the grey colors of the original picture are preserved.

An important theoretical result was demonstrated by replacing the grey line pattern by a uniformly colored line pattern. With this policy we could show that “equi-brightness” is an equivalence relation. In other words we could show, that the following for each monitor colors F and G holds:

The line pattern colored by F leads to minimal apparent motion on the background colored by G if and only if $B(F) = B(G)$.

The methode of teeming photometry enables the attribution of a unique brightness to a background whit a spatial double periodic pattern.

Using an inhomogene background with a spatial double periodic pattern and a spatial primitive periode which has an angle of view smaller than 3 degrees, one get a minimum of apparent motion of the grey line pattern too. At the one hand in this situation one can percept the inhomogenity by peripheral vision. On the other hand the brightness of the grey line pattern corresponds with the spatial average color of the pattern of the background.



That means for instance: If there are the colors $F_1 = (X_1, Y_1, Z_1), ..., F_n = (X_n, Y_n, Z_n)$ appearing in the primitive periode, and have the concerning areas the measures $m_1, ..., m_n$ then by teeming photometry there is measured the brightness of the color $F = (X, Y, Z)$ with CIE-coordinates

$$X = (m_1X_1 +\ldots+ m_nX_n)/(m_1 +\ldots+ m_n), \tag{6a}$$

$$Y = (m_1Y_1 +\ldots+ m_nY_n)/(m_1 +\ldots+ m_n), \tag{6b}$$

$$Z = (m_1Z_1 +\ldots+ m_nZ_n)/(m_1 +\ldots+ m_n). \tag{6c}$$

If there are only slight contrasts of luminancy in the primitive periode, then one can measure the average brightness even for bigger primitive periodes.

Completing Remarks.

Increasing the rate of presentation of the grey line pattern from 18 different patterns to 7 different patterns per second does not change the results.

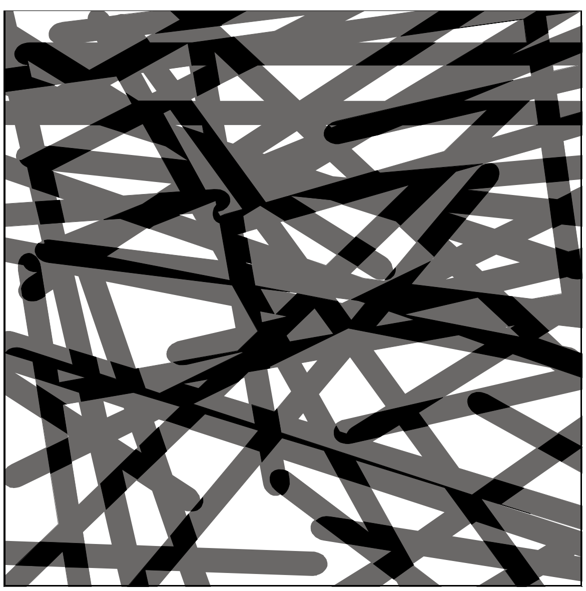
The perception of the apparent motion does not decrease by phenomenas of habituation in the relatively short time of matching one color.

In the situation of minimal apparent motion there is a precise perception of the hue of the background. Therefore in this situation we have photopic color vision.

Discussion of the perception of apparent motion occurring in teeming photometry.

In the case where background and line pattern have different brightnesses, it seems that some well-known effects are influencing the perception of apparent motion.

Due to the calculation of the line pattern in a certain point of the stimulus there can occur intervals in time with length 50 msec, 100 ms, 150 msec, … in which the color of that point is constant. The following picture shows a typical superposition of two successive line patterns.



The area, which is covered by lines with exactly one of the line patterns, is drawn grey.

The area, which is covered by both line patterns, is drawn black. One can remark that the grey area is substantially bigger than the black area. So the area, which is covered by lines in three successive line patterns, is negligible.

1. By the Broca Sulzer effect[6,11-18] flashes with a duration of 50 msec lead to a supplementary brightness perception. For flashes with a duration of 100 msec the Broca Sulzer effect is less prominent. Concerning the teeming experiment, the Broca Sulzer effect therefore should increase the differences of brightness. But by decreasing the rate of line patterns 18 per second down to 7 per second the Broca Sulzer effect appears no longer. For this situation we could not observe a change (decreasing) of brightness. Therefore the Broca Sulzer effect should not be prominent at all. Our standard rate of 18 line patterns per second is chosen, because it makes the brightness matching very comfortable.

2. The duration of the shortest flashes occurring in teeming-photometry are 50 ms.
For cone vision Bloch's law[19] of temporal summation is negligible because the duration of complete summation is about 20 msec. But for rods we have a duration of complete summation[20,21] of about 150 msec and therefore there could exists a decreasing effect for the perception of the apparent motion.

3. For flashes with duration between 50 msec and 200 msec the phi-phenomenon[22,23], as originally discovered by Wertheimer, leads to the perception of an apparent motion in the test field of the teeming experiment.

In the case of consistency of the brightness of background and line pattern there is induced no Broca Sulzer effect and the phi-phenomenon is not based on brightness differences. Therefore the percepted apparent motion is very week[24-30].

A rather strange discovery concerns the possibility for matching the average brightness of a inhomogeneous background by the teeming-experiment. In this situation there is no perception of the average brightness, but the observer has simultaneously the perception of the pattern of the background by peripheral vision and the vanishing of the apparent motion.

Color perception depends on a lot of determining factors like the ambiente of the presentation, the current condition and the molecular genetic disposition[31-46] of the observer. Therefore it is surprising to find no such blazing effects.

The individual differences in the brightness perception led to the introduction of the luminous efficiency function[47-54] $V(\lambda)$ by the CIE as a standard for spectral colors.

The experimental tool case was the flicker photometer.

By the widely consistency of the results based on flicker photometry and on teeming photometry and by the enlargement of the application area by the teeming photometry on peripheral motion vision there is given a further supplementary argument for the appropriation of the luminous efficiency function for technical purposes.

The prominent features of the teeming photometry compared with the flicker photometry are a easily realizable arrangement, the fast winning of the results and the superior degree of accuracy. Therefore in experiments on the monitor, which require an exact user adjustment of the personal specific brightness of colors, in future the teeming photometry should replace the flicker photometry.

It is conceivable to design a teeming photometer, which allows studying temporal truthfully constant stimulis too.

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